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Cournot Oligopoly Game-based Local Energy Trading considering Renewable Energy Uncertainty Costs

Yajun Zhang, Chenghong Gu, Xiaohe Yan, and Furong Li

Dept. of Electronic and Electrical Eng., University of Bath, Bath, BA2 7AY, U.K

Abstract—Facilitated by advanced information and communication technologies (ICTs), local energy trading develops rapidly, playing an important role in the energy supply chain. Thus, it is essential to develop local trading models and strategies that can benefit participants, not only stimulating local balancing but also promoting renewable penetration.

This paper proposes a new local energy trading decision-making model for suppliers by using the Cournot Oligopoly game, considering the uncertainty costs of renewable energy. Four types of representative energy providers are modelled, traditional thermal generation, wind power, photovoltaic (PV) power and electricity storage. The revenue of these technologies is extensively formulated according to their operation cost, investment cost, and income from selling energy. The uncertainty cost of renewable generation is integrated into the trading, modelled as a penalty for potential energy shortage that is derived from output probability distribution function (PDF). This trading model is formulated as a non-cooperative Cournot oligopoly game to enable energy suppliers to maximize their profits through local trading considering price. The response of the customer to energy price variations, i.e. demand elasticity, is also included in the model. A unique Nash equilibrium (NE) and optimum strategies are derived by the proposed Optimal-Generation-Plan (OGP) Algorithm. As demonstrated in a typical local market, the proposed approach can effectively model and resolve multiple suppliers' competition in local energy trading. It can work as a vehicle to facilitate the trading between various generation technologies and customers, realising local balancing and benefiting all market participants with enhanced revenue and reduced energy bills.

***Keywords* —**Cournot game, local energy trading, renewable energy, renewable uncertainty, price elasticity, energy storage, electric vehicles.

1. INTRODUCTION

From EIA statistics, the consumption of electricity worldwide increased three times from 7323BkWh to 21153BkWh in the past 35 years [1]. The continuous increase of demand brings a huge challenge to the electricity industry in generating, transporting, and distributing energy. Currently, thermal power stations are still the major producer of global electricity, producing over 60% of electricity [2]. By contrast, distributed energy resources (DERs), driven by renewable technologies, such as wind power and solar power, are encouraged by governments in recent years [3], developing at a fast rate. Furthermore, advanced technologies, such as energy storage, are widely used to support renewable energy in mitigating their intermittence with up to 2.2GW worldwide capacity in 2018 [4]. By taking advantage of fluctuating electricity prices, energy storage owners could buy energy at cheaper prices and sell it at higher prices locally with equipment of battery, electric vehicles (EVs), etc.,[5]. This arbitrage from energy storage also fundamentally change traditional electricity procurement models.

Incentivized by policies, EVs techniques develop fast in the last decade mainly due to the concern of environmental issues [6]. EV can be used as a means of clean transport but also can be regarded as movable energy storage, which can have many flexibilities [7]. Nowadays, Electric vehicles (EVs) are widely used in the transportation sector, which has broad application prospects in the smart grid. Particularly, if EVs are properly managed, they can provide many services to energy markets and network operators. For example, paper [8] makes use of it to improve the power quality of the grid that minimizes voltage sags with the discharge power from EV. Paper [9] investigates the strategies of EV charge/discharge management in the smart grid to benefit EV owners. Paper [10] present the profitable model for EV owners through optimization of the EV charge and discharge plan. However, as the capacity of a single EV is too small to participate in markets, the concept of EV aggregator has been proposed to manage a large number of EVs. As an agent, EV aggregators can manage EV charging and discharging to participate in the energy market and provide services to networks, thus helping maximize EV owners' profits [11]. In this paper, EV aggregator is modelled as flexible energy storage to participate in the local market and use the proposed optimal strategies.

With the high penetration of DERs in distribution systems, local energy trading is proposed to facilitate local balancing to promote renewable energy penetration and reduce energy costs for customers. In recent years, many studies have focused on the trading mechanism in local energy markets. In paper [12], a bidirectional trading mechanism is designed with the utilization of electric vehicles (EVs). By using robust game theory, authors present a collaborative trading model to maximise the whole system welfare and develop a non-collaborative trading model to help prosumers improve their profits. In paper [13], the authors propose a contract scheme, which provides small-scale electricity suppliers and consumers the chance to participate in direct energy trading for revenue maximisation under asymmetric information energy trading. Paper [14] presents an approach to form an optimal coalition of heterogeneous DERs in a commercial virtual power plant (VPP) based on weekly bilateral contracting. With this optimal portfolio of DERs, the VPP managers could maximize their profits. These studies contribute to involving DER into the local energy market, but the uncertainties of renewable DER have been neglected in trading. The previous work on local energy trading mostly models treats renewable energy as controllable, which is not practical. This shortcoming makes these models difficult to achieve desired results in practical application.

The uncertainty of renewable energy constrains its trading, as there is a high risk of it not being able to deliver contracts. Recently, many studies have investigated renewable uncertainties. Paper [15] proposes an optimization algorithm to maximize the utilization of renewable energy sources and plug-in EVs. Renewable energy uncertainties are companied by a set of valid scenarios. Paper [16] presents an optimal operation planning for an isolated microgrid. Taking the consideration of renewable uncertainties, the Gaussian distribution is applied to uncertainty modelling. In paper [17], a multi-scale wind model is developed considering uncertainty for decision making in the operational planning of microgrid. In summary, the uncertainty researches in these papers only focus on the physical level. To apply it in energy trading, the conversion between the physical power flow and cash flow on the economic level is required to build.

In modelling and resolving local energy trading, game theory is widely adopted to formulate trading schemes and derives optimal strategies for profit maximisation in Nash equilibrium (NE), where no player has any incentives to change strategies because they cannot benefit more by changing when others retain their strategies [18]. Paper [19] proposes an event-driven local energy trading for microgrids. The Stackelberg game is adopted and the desired trading scenario is derived by Stackelberg Nash equilibrium. In paper [20], the authors use the Bayesian game model to formulate the schedule of EV energy consumption in bidirectional energy trading. Their optimal scenarios are obtained by the Bayesian Nash equilibrium point. Apart from these models, the Cournot model, with duopoly and oligopoly models, is widely used. Paper [21] uses the Cournot oligopoly model for a three-player competition in a transmission-constrained system considering the non-constant marginal cost and derives the equilibrium point. In paper [22], the agents' profit-maximizing behaviours in the wholesale market are formulated and solved. Paper [23] applies the Cournot model to analyse the nodal prices and transmission constraints in a multimarket. In paper [24], Cournot oligopoly model is adopted for the imperfect competition among suppliers considering the energy arbitrage.

It can be seen that existing studies on the Cournot game applied to the electricity market focus on the competition among players, but they ignore the differences in the production and operation of various technologies, renewable, storage, and traditional suppliers. In energy markets, multiple generators such as thermal power stations, wind turbines, photovoltaic (PV) panels, etc., have very different characteristics, which will lead to various cost functions and trading strategies. Thus, it is essential to develop local energy trading models that can authentically reflect both the physical and economic features of various energy suppliers, particularly for the intermittent generation which has limited controllability and high uncertainties. In summary, with increasing DERs in distribution networks, local energy trading can help them be consumed locally, but existing work has obvious weaknesses, which are: 1) the physical differences and revenue functions between different generators are not well modelled; 2) the uncertainties and uncertainty costs of renewables are not considered in energy trading.

To resolve the aforementioned drawbacks, this paper develops a novel local energy trading model and derives the optimal trading strategies for suppliers to ensure their benefits. The Cournot oligopoly game model is adopted to formulate supplier competition. Four types of typical power suppliers, thermal power station, wind farm, solar PV, and energy storage, are respectively formulated to build their business models according to investment, operation, and technologies. In their business models, profits are decided by trading prices and production quantity, and the costs involve investment, operation, and uncertainty. Particularly, the uncertainty cost of renewable generation is integrated into the trading, modelled as a penalty for potential power shortage derived from the output probability distribution function (PDF). An Optimal-Generation-Plan (OGP) algorithm is proposed to balance the conflict between market price and power generation. The Nash equilibrium (NE) in this Cournot oligopoly game is derived by the OGP algorithm and the optimal strategies at the NE for all suppliers are obtained to ensure their maximum profits.

The main contributions of this paper are: i) it designs a novel local non-cooperative trading model considering renewable generation and energy storage; ii) it extensively models the uncertainty costs for renewable generation in local energy trading; iii) the features of various generations are modelled in local energy trading with a Cournot Oligopoly game for the first time; iv) it solves the model with the proposed OGP algorithm, which is effective in finding the equilibrium and optimal trading strategies for suppliers.

The rest of the paper is organized as follows: Section 2 illustrates local energy trading and discusses the price elasticity of demand. Section 3 presents the models for four different types of suppliers (thermal, wind, PV, storage) and their costs and revenues. Section 4 proposes a supplier competition model formulated by Cournot oligopoly and provides the solution. The case study provided in Section 5 proves the effectiveness of the proposed approach and conclusions are drawn in Section 6.

2. LOCAL ENERGY TRADING AND DEMAND ELASTICITY

2.1 Local Energy Trading

Promoted by government policies and facilitated by new technologies, DERs are boosted in recent years, mainly in terms of micro wind turbines and rooftop solar photovoltaic panels. These small-size power sources at the customer's side effectively help them save energy bills and empower the controllability of energy consumption [25]. Thus, the power supply could be realized at the local level, which promotes the development of local energy trading. Local energy trading means that in a physically close area, an energy market is designed so that suppliers and prosumers with excessive energy can sell it in the market [26]. For customers, they can buy the cheaper energy needed from those multiple providers in the local energy market, rather than only the retailers from the central market. At present, there are diverse participated suppliers in the local market, including not only conventional power plants but also renewable energy, such as PV and wind. In addition, EVs and storage technologies induce a new business mechanism in local market trading. For example, energy storage owners and EV aggregators can make profits by energy arbitrage in the local market. Those various suppliers all compete for the market share to serve end customers [27]. As shown in Fig. 1, customers could purchase their energy from both the main grid and local energy market to find an optimal strategy to save bills. From the perspective of the energy price, they are generally served with lower prices by the local energy market [28].

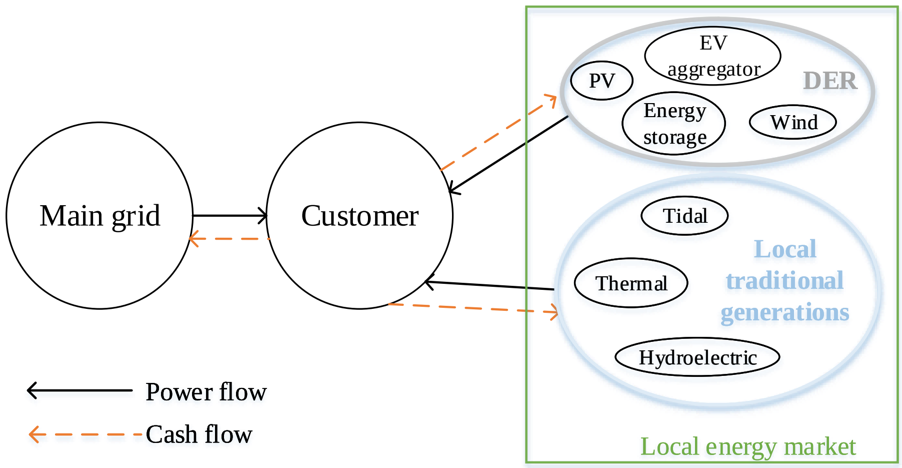

Fig. 1. Local energy trading

In comparison to centralized energy trading, local energy trading is more flexible with small volume, occurring in the distribution system [29]. Hence, it is capable to promote DERs which is excluded because of its small scale and intermittency in the centralized electricity market [30]. With the utilization of smart grid technologies and information and communication technologies (ICT), the power from DERs can be traded solely or integrated by agents in the local energy market, which benefits end-customers with low energy prices and reducing the waste of renewable energy. Furthermore, local energy trading effectively relieves the heavy burden on the main grid and potentially avoids the expensive grid expansion [31]. On the other hand, there are potential risks induced by renewable uncertainties. Thus, a reasonable trading mechanism is required to provide an appropriate trading platform for energy suppliers and consumers to realize local balancing, which benefits both sides.

2.2 Price Elasticity of Demand

From the market perspective, the real demand fluctuates above and below the scheduled demand because it is affected by energy prices. In the local trading, the low electricity price will motivate consumers to increase demand and on the contrary, they will reduce their unnecessary usage to save bills when the price is high. It reflects the price elasticity of demand.

The price elasticity of demand is defined as the slope of the demand curve [32], representing the proportionate change in demand to the proportionate change in energy price:

$$\varepsilon = \frac{\Delta D/D_0}{\Delta\delta/\delta_0} \tag{1}$$

where D is the demand in the local energy market; ΔD is the change in the quantity of demand; $\Delta\delta$ is the corresponding change in energy price; $(D_0,\, \delta_0)$ is the reference point chosen in the demand curve.

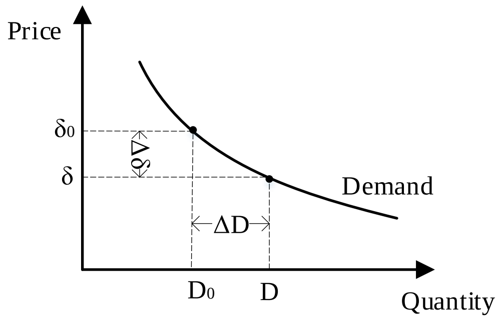

Fig. 2. Price elasticity of Demand

In the energy market, there is an interplay between demand, supply, and energy price. Thus, the demand-supply relationship will affect the energy price in the local market and trading. From the perspective of demand, it consists of two parts: rigid demand and elasticity demand. The rigid demand depends on the necessary consumption, normally being a certain value for the period. While the elasticity demand represents the consumption without the requirement of the specific duration, affected by the energy price. The complicated relationship can be concluded as a constraint, which is linearized for simplicity, that:

$$D_0 + \varepsilon(\delta - \delta_0) - \sum P = 0 \tag{2}$$

where the $\sum P$ is the sum of the power output of each participated supplier, representing the total generation of the local energy market during hourly trading.

3. Supplier Business Model

In this section, four types of electricity suppliers, thermal power station, wind power generation, PV power generation, and electricity storage at the local level, are analysed to build their mathematical formulations for hourly trading. Their operational costs are quantified according to generation quantities and technologies. Their cost functions and revenue functions are also derived respectively.

From the perspective of suppliers, their costs are decided by three key time factors: initial investment, operation, and trading. In the initial building period, there is a setup capital. The initial investment covers the cost of building the plant, devices installation, obtaining related permission, etc. Then, during operation, there is operational cost, including the generation cost, the cost for operating and maintenance. In the trading period, due to the intermittency of renewable energy, there is an extra uncertainty cost for renewable suppliers to secure energy transactions.

3.1 Investment Recovering Cost

The investment cost varies, depending on generation technologies. For thermal power stations, the initial investment covers the location fee, cost of devices (boiler, pre-heater, cooling system, etc.), cost of safety equipment, cost of various government permissions and the cost of building up the power plant. Although a series of such expenditures require a large number of initial costs, the investment recovering cost distributed to unit power is not very high with corresponding large-scale generation.

The initial investment plays generally a major role in the total cost and recovering the investment cost would take years. The investment recovering cost (IRC) related to the power output can be formulated as:

$$IRC(P) = \frac{C_{initial} * (1+d)^{N_{plan}}}{N_{plan} \times E_{annual}} P \tag{3}$$

where $C_{initial}$ is the initial investment of supplier; N_{plan} is the number of years to recoup the initial capital in the plan; d is the discount rate; E_{annual} is the total annual power output of the suppliers; P is the hourly power output.

3.2 Operational Cost

During the operation period, there is related operational cost.

1) Thermal power station

The thermal power station converts the heat energy to electric power, whose operation cost is determined by fuel cost, incremental cost, input/output, and heat rate. Its generation cost is formed as a quadratic equation [33]:

$$OC_t(P_t) = aP_t^{\,2} + bP_t + c \tag{4}$$

where P_t is the power output of power station; a, b and c are the fuel cost coefficients.

2) Wind power generation

In comparison to the traditional thermal power plant, wind power is free, clean, and reproducible, saving the expenditure on material procurement. Meanwhile, wind power is intermittent and thus there are balancing costs induced due to the errors in hourly forecasting. According to paper [34], the operational cost of the wind farm is formulated by:

$$\begin{aligned} OC_w(P_w) &= \int_0^P WPUIC(w)dw + C_{onst} \\ &= \alpha_w P_w^{\,2} + \beta_w P_w + \gamma_w \end{aligned} \tag{5}$$

where $WPUIC$ is the wind power uncertainty incremental cost which qualifies the extra balancing cost for increasing unit wind power generation; C_{onst} is the cost for balancing wind power deviation; P_w is the power output of wind farm; α_w, β_w, and γ_w are adjustable coefficients.

3) PV power generation

Similar to wind power, the PV system is cost-benefit and clean without carbon emission. There are annually fixed operation cost and maintenance fee, which dominate the operation cost [35], modelled as follows:

$$OC_{PV}(P_{PV}) = \frac{O_{PV} + M_{PV}}{E_{annual}} P_{PV} \tag{6}$$

where P_{PV} is the PV power output; OPV is the annual operation cost; M_{PV} is the annual maintenance fee.

4) Electricity storage

The energy storage makes money through energy arbitrage by making use of the fluctuations in electricity prices. For battery owners, storage will charge the battery during the off-peak periods with low prices and sell them back to the market at a high price in peak time. According to Schumacher's battery model, there exist energy losses during the charging and discharging process [36], which will induce extra energy consumption. From the perspective of generation cost, it contains the cost of charging the battery, proportional operation cost, and fixed maintenance fee. The mathematical formulation for the operation cost is defined as [35]:

$$OC_s(P_s) = \delta_{in} \frac{P_s}{(1-\eta_d)} + \delta_{in} \frac{P_s}{(1-\eta_d)} \eta_o + M_s \tag{7}$$

where P_s is the power output of storage supplier; δ_{in} is the unit purchase price for electric power; η_o is the weighting coefficient of the operation cost, η_d is the battery deterioration rate of energy; M_s is the maintenance fee.

5) EV aggregator

For EV aggregators, they can also participate in the local trading to procure energy when charging and sell energy when discharging on behalf of EVs. By using appropriate incentives to manage EVs charging and discharging behaviours, there is considerable profit for EV aggregators. It is assumed that the EV aggregator manages m EVs in its area. Thus, the capacity of EV aggregator can be derived as the sum of available storage space for each EV:

$$P^c = r \cdot \sum_{i=1}^{m} ca_i - D_i \tag{8}$$

where r is the charger efficiency; ca_i is the capacity of EV i; D_i is the demand of EV i.

Based on energy arbitrage, the operation cost of EV aggregator mainly depends on electricity cost and battery degradation cost [37], represented as:

$$OC_{EV}(P_{EV}) = \delta_c \frac{P_{EV}}{r} + DE * k \tag{9}$$

where k is the number of manageable EVs for discharging; δ_c is the electricity price during the charging; DE is the battery degradation cost for an EV.

3.3 Uncertainty Cost

1) Renewables

Renewable generation commonly relies on natural sources, such as wind power, solar power, kinetic energy from the river [38]. They are clean but also un-dispatchable. Due to this feature, the uncertainties of renewable bring the challenge to local energy trading. To handle this problem, there is the extra cost related to renewable uncertainties to guarantee that renewable suppliers can provide agreed energy. If the real output generation cannot satisfy the amount required in the transaction, suppliers need to pay the penalty for the shortage. In this case, the shortfall will be purchased from the ancillary market at a high price. Therefore, there is a penalty applied to the potential shortage part, whose expectation can be expressed by the product integral of the shortage and its corresponding probability. The renewable uncertainty cost (UC) can be formulated as:

$$UC(P) = \delta_{penalty} \int_0^P f(P^+)(P - P^+) dP^+ \tag{10}$$

where P^+ is the real power output; P is the planned power output in energy transaction; $f(P)$ is the PDF of renewable power output; $\delta_{penalty}$ is the unit electricity price in the ancillary market.

The PDF of renewable power output is derived by Kernel density estimation (KDE) according to historical data [39], as follows:

$$f_h(x) = \frac{1}{nh} \sum_{i=1}^{n} K(\frac{x - x_i}{h}) \tag{11}$$

where n is the size of samples; h is the bandwidth between samples; K is a non-negative function referred to as the kernel satisfying $\int K(x)dx = 1$.

For the wind power generation, the kernel function could be modelled by Cauchy distribution [40]:

$$K\left(\frac{x - x_i}{h}\right) = \frac{1}{\pi}\left[\frac{\gamma}{\left(\frac{x - x_i}{h} - x_0\right)^2 + \gamma^2}\right] \tag{12}$$

where x_0 is the peak location parameter; γ is the half-width at the half-maximum, indicating the scale.

For the PV power generation, the kernel function could be modelled by normal distribution [41]:

$$K\left(\frac{x - x_i}{h}\right) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\left(\frac{x - x_i}{h} - \mu\right)^2}{2\sigma^2}\right) \tag{13}$$

where μ is the expectation of the distribution at the same place as median; σ is the standard deviation; σ^2 is the variance of the distribution.

2) EV aggregator

EV aggregators can also face uncertainty as the manageable energy power discharging is affected by EV owner behaviours, which is beyond control. Therefore, the uncertainty is dominated by the available discharging EVs number k. The real energy output can be represented as:

$$P_{EV}^+ = soc \cdot r \cdot k^+ \tag{14}$$

where soc is the state of charge manageable EVs that can be used for discharging; k^+ is the number of real arrived EVs.

By submitting (14) into (10), EV aggregator uncertainty cost can be calculated as:

$$UC_{EV}(P_{EV}) = \delta_{penalty} \int_0^k f(k^+)(P_{EV} - soc \cdot r \cdot k^+) dk^+ \tag{15}$$

3.4 Revenue Function

According to generation technologies, their cost functions are proposed and revenue functions as follows.

1) Thermal power station

The thermal power station is fuelled by natural gas or coal, making it highly controllable. Because of this feature, there is no uncertainty cost for the thermal power station. Its cost mainly includes investment cost and operational cost, which is derived as follows in (16). The first part is operation cost derived in (4) and the second part is investment cost calculated in (3).

$$C_t(P_t) = OC_t(P_t) + IRC_t(P_t) \tag{16}$$

The revenue for the thermal power station is the income from selling energy (unit price times output) minus the total cost, which is as follows:

$$R_t(P_t) = \delta P_t - C_t(P_t) = \delta P_t - OC_t(P_t) - IRC_t(P_t) \tag{17}$$

where δ is the unit energy price in the local market.

By substituting (3) and (4) into (17), the final revenue function for the thermal power station can be obtained as follows:

$$R_t(P_t) = \delta P_t - \left(a{P_t}^2 + bP_t + c + \frac{C_{initial} * (1+d)^{N_{plan}}}{N_{plan} \times E_{annual}} P_t \right) \tag{18}$$

2) Wind power generation

Generally, large size wind powers plants perform better in controllability and the small size is more easily affected by the weather. Thus, from the perspective of cost, apart from the initial investment cost in (3) and operational cost in (5), the uncertainty cost in (10) in local trading is essential to be considered. The cost function of the wind farm is formulated by the sum as follows:

$$C_w(P_w) = OC_w(P_w) + IRC_w(P_w) + UC_w(P_w) \tag{19}$$

The revenue function can be derived through the income minus its total cost, which is as follows:

$$R_w(P_w) = \delta P_w - C_w(P_w) = \delta P_t - OC_w(P_w) - IRC_w(P_w) - UC_w(P_w) \tag{20}$$

Replaced by (3), (5) and (10) into (20), its final revenue function can be represented as follows:

$$\begin{aligned} R_w(P_w) = \delta P_w - \Bigg(& \alpha_w {P_w}^2 + \beta_w P_w + \gamma_w + \frac{C_{initial} * (1+d)^{N_{plan}}}{N_{plan} \times E_{annual}} P_w \\ & + \delta_{penalty} \int_0^{P_w} f(P_w^+)(P_w - P_w^+) dP_w^+ \Bigg) \end{aligned} \tag{21}$$

where P_w^+ is the wind power output in the practical generation; $f(P_w^+)$ is the PDF of wind power output which is formulated by Cauchy distribution.

3) PV power station

PV power generation also faces the challenge of uncertainties, whose cost function is expressed by the sum of the uncertainty cost in (10), the initial investment cost in (3) and operational cost in (6) as:

$$C_{PV}(P_{PV}) = UC_{PV}(P_{PV}) + IRC_{PV}(P_{PV}) + OC_{PV}(P_{PV}) \tag{22}$$

Thus, the profit of the PV generator can be proposed by subtracting its cost from the sale income:

$$R_{PV}(P_{PV}) = \delta P_{PV} - C_{PV}(P_{PV}) = \delta P_{PV} - UC_{PV}(P_{PV}) - IRC_{PV}(P_{PV}) - OC_{PV}(P_{PV}) \tag{23}$$

By substituting (3), (6) and (10) into (23), the final revenue function can be derived as follows:

$$\begin{aligned} R_{PV}(P_{PV}) = \delta P_{PV} - (& \frac{C_{initial} * (1+d)^{N_{plan}}}{N_{plan} \times E_{annual}} P_{PV} + \delta_{penalty} \int_0^{P_{PV}} f(P_{PV}^+)(P_{PV} - P_{PV}^+) dP_{PV}^+ \\ & + \frac{O_{PV} + M_{PV}}{E_{annual}} P_{PV}) \end{aligned} \tag{24}$$

where P_{PV}^+ is the PV power output in the practical generation; $f(P_{PV}^+)$ is the PDF of PV power generation output which is formulated by the normal distribution.

4) Electricity storage

Different from renewable generation, energy storage is limited by the capacity of batteries. In terms of the total costs, it only has the operation cost in (7) and investment cost in (3). The mathematical formulation of the cost function is:

$$C_s(P_s) = OC_s(P_s) + IRC_s(P_s) \tag{25}$$

Then, according to its total income and total cost, the revenue function is deduced by this difference as:

$$R_s(P_s) = \delta P_s - C_s(P_s) = \delta P_s - OC_s(P_s) - IRC_s(P_s) \tag{26}$$

By substituting (3) and (7) into (26), the final revenue function can be obtained as follows:

$$R_s(P_s) = \delta P_s - \left[\delta_{in}\frac{P_s}{(1-\eta_d)}(1+\eta_o) + M_s + \frac{C_{initial} * (1+d)^{N_{plan}}}{N_{plan} \times E_{annual}} P_s\right] \tag{27}$$

5) EV aggregator

For EV aggregators the total cost contains investment cost in (3), operation cost in (9) and uncertainty cost in (15), represented as:

$$C_{EV}(P_{EV}) = IRC_{EV}(P_{EV}) \ + OC_{EV}(P_{EV}) + UC_{EV}(P_{EV}) \tag{28}$$

Hence, the revenue function is obtained by subtracting the total cost from its total income as follows:

$$R_{EV}(P_{EV}) = \delta P_{EV} - C_{EV}(P_{EV}) = \delta P_{EV} - IRC_{EV}(P_{EV}) \ - OC_{EV}(P_{EV}) - UC_{EV}(P_{EV}) \tag{29}$$

By substituting (3), (9) and (15) into (29), the final revenue function can be obtained as follows:

$$\begin{aligned} R_{EV}(P_{EV}) = \delta P_{EV} - \Bigg[&\frac{C_{initial} * (1+d)^{N_{plan}}}{N_{plan} \times E_{annual}} P_{EV} + \delta_c \frac{P_{EV}}{r} + DE * k \\ &+\delta_{penalty}\int_0^k f(k^+)(P_{EV} - soc \cdot r \cdot k^+)dk^+\Bigg] \end{aligned} \tag{30}$$

4. SUPPLIER COMPETITION IN THE LOCAL ENERGY MARKET BY COURNOT OLIGOPOLY

4.1 Supplier Competition Model

In the local energy market, multiple suppliers compete for market sharing. This supplier competition is formulated by the Cournot oligopoly model. The participated suppliers are defined as $\omega=\{1, 2,\ldots, n\}$, which independently determines their own production to achieve profit maximization. According to the revenue functions in (13), (15), (17) and (19), it can generally be concluded with the form as:

$$R_i(P_i) = \delta P_i - C(P_i);\ \forall i \in \omega \tag{31}$$

where P_i is the power output of the supplier i; $C(P_i)$ is the cost function of supplier i; ω is the set of suppliers.

In this revenue function, the energy market price δ is affected by the total generation with the constraint in (2). From the demand-supply curve, the price will decrease with the growing supply. Therefore, the suppliers need to find an equilibrium for their generation to maximize their revenues.

$$\max R_i(P_i) = \left(\frac{\sum_{i\in\Omega} P_i^* - D_0}{\varepsilon} + \delta_0\right) P_i^* - C(P_i^*);\ \forall i \in \omega \tag{32}$$

$$\text{s.t. } P_i^{min} \le P_i \le P_i^c$$

where P_i^* is the optimal production of supplier i corresponding to its maximum profit; P_i^{min} is the low boundary of the generation for supplier i; P_i^c is the generation capacity of supplier i.

4.2 The Optimal-Generation-Plan Algorithm

The proposed OGP algorithm is to find the Nash equilibrium point in the Cournot game model, where the optimal generation plan is derived. Every supplier can obtain the maximum revenue as long as they follow this generation strategy.

According to the supplier business model, their revenue functions all can be written in the following form generally.

$$R_i(P_i) = \delta P_i - (G_i P_i^2 + H_i P_i + K_i);\ \forall i \in \omega \tag{33}$$

where G_i, H_i, K_i are the constant depending on the cost coefficients of supplier i.

The unit energy price can be substituted as a function of all participants' production quantities in (2). Then, the revenue function could be rewritten as:

$$R_i(P_i) = \left(\frac{1}{\varepsilon} - G_i\right) P_i^2 + \left(\frac{\sum_{n\neq i}^{n} P_i - D_0}{\varepsilon} + \delta_0 - H_i\right) P_i - K_i \tag{34}$$

The maximum profit exists in this quadratic function when the partial derivate equals to zero [42], given when the following constraint satisfied:

$$2\left(\frac{1}{\varepsilon} - G_i\right) P_i + \frac{\sum_{n\neq i}^{n} P_i}{\varepsilon} = \frac{D_0}{\varepsilon} + H_i - \delta_0 \tag{35}$$

For the market with n participated suppliers, the whole system can be modelled as:

$$\begin{bmatrix} 2(\varepsilon^{-1} - G_1) & \varepsilon^{-1} & \cdots & \varepsilon^{-1} \\ \varepsilon^{-1} & 2(\varepsilon^{-1} - G_2) & \cdots & \varepsilon^{-1} \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon^{-1} & \varepsilon^{-1} & \cdots & 2(\varepsilon^{-1} - G_n) \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{bmatrix} = \begin{bmatrix} D_0\varepsilon^{-1} + H_1 - \delta_0 \\ D_0\varepsilon^{-1} + H_2 - \delta_0 \\ \vdots \\ D_0\varepsilon^{-1} + H_n - \delta_0 \end{bmatrix} \tag{36}$$

The optimal model can be solved in the Jacobian matrix by the Newton method [43]. In addition, the suppliers' generation capacity is set as the constraint in the optimization model.

4.3 Optimal Generation Strategy

In this multi-supplier trading, the proposed method is implemented in three stages in Fig. 3: identifying the participated suppliers, creating mathematical formulations, and determining generation strategies by OGP algorithm:

- The information about all market suppliers and demand is identified.
- It is to build suppliers' profit formulations. In the supplier model, their generation costs are formulated by two categories:
  - i) For the normal suppliers whose generation is controllable, their costs include two parts: investment cost and operational cost.
  - ii) For the renewable generation, the uncertainty cost should be considered in energy trading. Besides the investment cost and operational cost, the uncertainty cost is included to cover the potential shortage of their supply, developed according to the PDF of the practical generation.
- Based on the analysis of cost, the suppliers' revenue functions are formulated.
- Finally, the optimal strategy for each supplier is derived to help them maximize revenue by using the OGP algorithm.

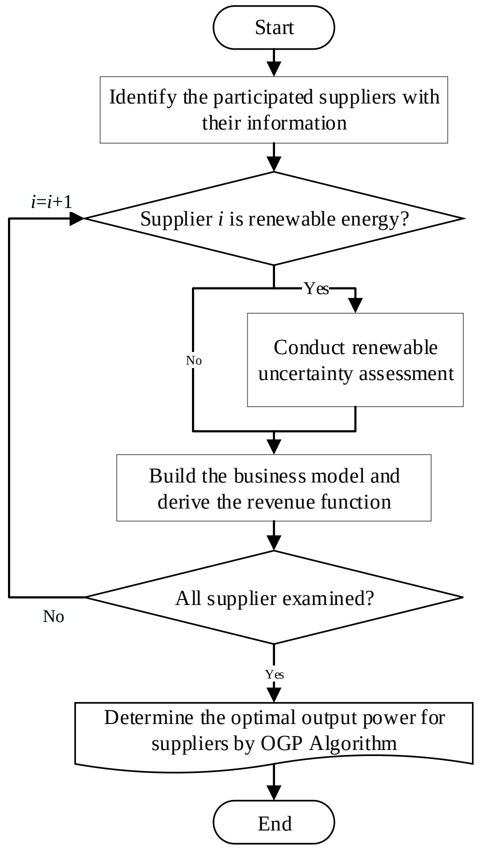

Fig. 3. Flowchart of the proposed method

5. CASE STUDY

In this section, a case study is provided to illustrate the proposed mechanism and designed models on hourly local energy trading. With the implementation of the OGP algorithm, the optimal generation strategies for suppliers can be derived at the NE point in the Cournot game model that helps them gain maximum profits.

There are four typical electricity suppliers investigated in this case study, which are thermal generation, wind power, PV power, and storage respectively. According to their generation technologies, investment cost, capacity constraints, and related operational cost coefficients are summarized in Table I.

TABLE I

FOUR SUPPLIERS' INVESTMENT COST, CAPACITY, AND OPERATIONAL COST COEFFICIENTS

Supplier	P^{min} (MWh)	P^{c} (MWh)	Investment Recovering Cost (£/MWh)	Operational Cost Coefficients		
Thermal	4	20	5.8	a =0.0087	b =13.3	c =81
Wind	5	20	7	α_w=0.002	β_w =10.4	γ_w =50
PV	0	3	15.7	O_{PV} = £15000	M_{PV} =£10000	E_{annual} =5000MWh
Storage	0	6	6.3	δ_{in} = £16/MWh	η_o = 5%	η_d = 10%

In order to ensure the fairness of the trading, the uncertainty cost is charged for wind and PV power. For the wind farm, the PDF of wind power output is followed by the Cauchy distribution [28], set as x_0=15 and γ =2. For the PV generation, the output is modelled by the normalized distribution [29], set as μ =2 and σ

=0.5. There is a penalty cost charged to cover the possible shortage in proportion to their PDFs of generation. The power shortage will be purchased from the ancillary market with a supposed price $\delta_{penalty}$= £35/MWh.

5.1 Optimal Strategies

In this case, the small local energy market is composed of these four suppliers with the price elasticity $\varepsilon = -2$ and the reference point for the market is set as $D_0 = 28\text{MWh}$ and $\delta_0 = £32/\text{MWh}$. In an-hour period, by using the OGP algorithm, the optimal output power (P^*) of suppliers with maximum revenues are derived, given in TABLE II. The iteration process is shown in Fig. 4.

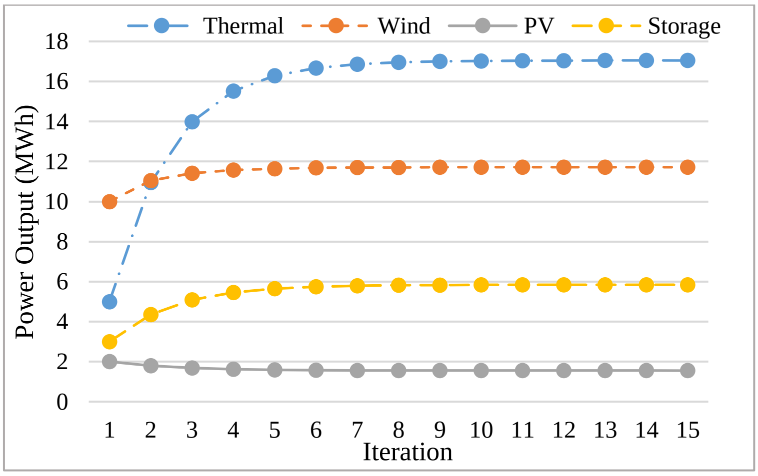

Fig. 4. Iteration of the OGP algorithm

TABLE II

OPTIMAL GENERATION STRATEGY WITH CORRESPONDING MAXIMUM REVENUES

Supplier	Thermal	Wind	PV	Storage
P*(MWh)	17	11.5	1.6	5.8
Max revenue (£)	67	58	9	15

Table II shows the NE point in this market. Following this generation strategy, each supplier could maximize its revenue. Their revenues and the validation of their optimal strategies are demonstrated in Fig.5.

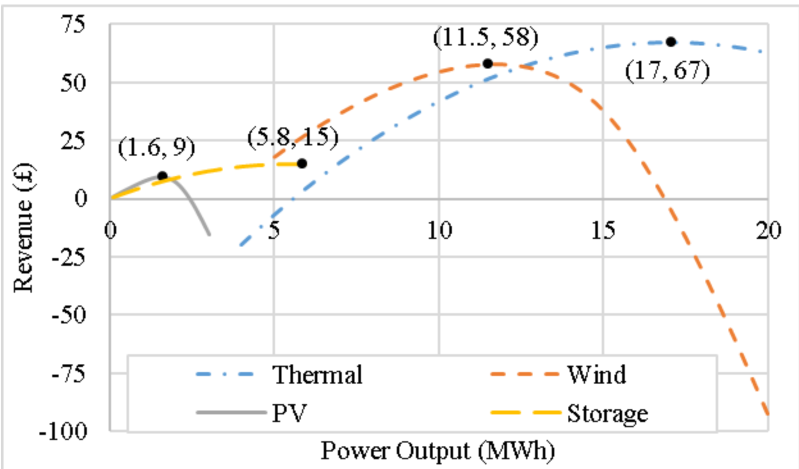

Fig. 5. Revenue for four suppliers when the other three suppliers all follow the optimal generation.

For the thermal power station, the curve "Thermal" demonstrates the results of: $\{P_t, P_{\mathrm{w}}^*, P_{\mathrm{PV}}^*, P_{\mathrm{s}}^*\}$, which reflects the change of its profit on the condition that the other suppliers follow the optimal generation strategies. Initially, there is a loss when production is under 5.6MWh because the earning is too low to cover its generation cost. Then it becomes profitable with the growing output. This revenue curve is a convex curve and the peak is arrived at 17MWh with the maximum profit of £67, matching the result of the OGP algorithm. For the wind farm, with the growing production, its revenue increases to the maximum point £58 at 11.5MWh, and then gradually falls due to the expensive uncertainty cost. When more wind power is required to supply, there is a growing potential risk accompanied. It clearly shows in the diagram that when the planned output power is over 16.8MWh, there is a deficit due to expensive uncertainty cost above £84. For PV, its maximum revenue is £9 achieved by 1.6MWh, corresponding to the calculated optimal strategy. It is profitable when its generation is below 2.4MWh. As renewable energy, it also faces expensive uncertainty cost when the high quantity is required. Thus, it would face an economic loss after 2.4MWh. For storage, the profitability of energy arbitrage is based on the price difference. Clearly shown in Fig. 5, its maximum profitable point is 5.8MWh in generation with an earning of £15. It is clear that the generation quantities at their maximum profit points for these four suppliers all corresponding to the NE point derived by the OGP algorithm. It verifies the effectiveness of the proposed approach.

5.2 Sensitivity Analysis

5.2.1 Uncertainty of renewables

The uncertainty of renewable energy determines its performance in local trading, thus affecting the profits. For the wind farm, the uncertainty is reflected by the PDF of its power output. Specifically, it depends on two parameters in the Cauchy distribution of PDF: location parameter x_0 and scale parameter γ.

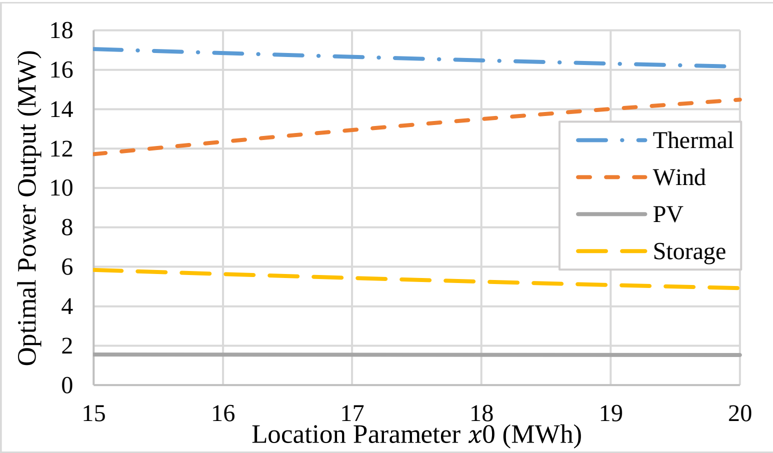

Fig. 6. Optimal power output for suppliers in different peak location 15, 16, 17, 18, 19, and 20 of wind PDF when γ =2

From the perspective of output PDF, x_0 is the peak location indicating the situation with the highest probability, whose impact is displayed in Fig. 6. When the peak of PDF moves from 15MWh to 20MWh, the wind plant has a larger generation output according to its optimal strategy, from 11.7MWh to 14.5MWh. On the contrary, the other three suppliers would slightly reduce their generation, 0.9MWh for thermal, 0.1MWh for PV, and 0.9MWh for storage to optimize their profits. The increase in x_0 enhances the supply capacity of the wind farm, which leads to an increase in P_w*. Due to the enhanced generation capacity of the wind farm, there is a small decrease in the optimal output for other suppliers. In conclusion, the larger x_0 the better it is for the wind farm.

As far as the scale parameter is concerned, γ is the half-width at the half-maximum of PDF. In other words, the larger the value of γ, the curve of PDF is more flattening. Fig. 7. reflects the impact of γ when x_0=15MWh. It can be seen that wind has the biggest change because it is directly affected by γ. Its optimal output power shows a 1.7MWh-decrease from 12.8MWh to 11.1MWh, when its PDF scale grows from 1 to 3. Meanwhile, the optimal output for the other three suppliers all shows a slight increase, 0.5MWh for thermal, 0.1MWh for PV, and 0.6MWh for storage respectively. The bigger γ means the more even PDF of wind plant output, which would bring more uncertainty cost for wind power. Thus, the optimal generation strategy is to reduce output in compensation for the increased risk of uncertainty cost. To conclude, the wind farm has a better performance with a smaller γ.

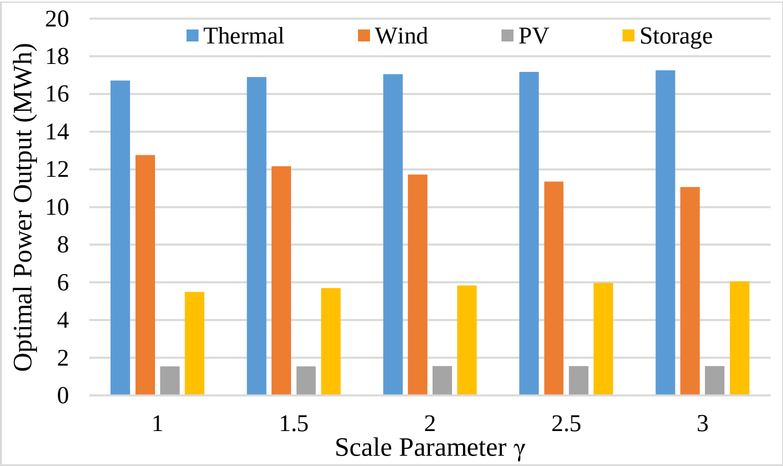

Fig. 7. Optimal power output for suppliers in different scale 1, 1.5, 2, 2.5 and 3 of wind PDF when x_0=15MWh

5.2.2 Uncertainty cost

Although the uncertainty cost is a burden to renewable energy, it is significant to secure the fairness of energy trading and avoid the potential risk of power imbalance. For the wind farm, its generation cost is illustrated in Fig. 8. Clearly, the generation cost with uncertainty cost exponentially increases with growing output. When more power is planned to trade, the higher the risk of the shortage between practical generation and plan, and the higher uncertainty cost is induced at the same time.

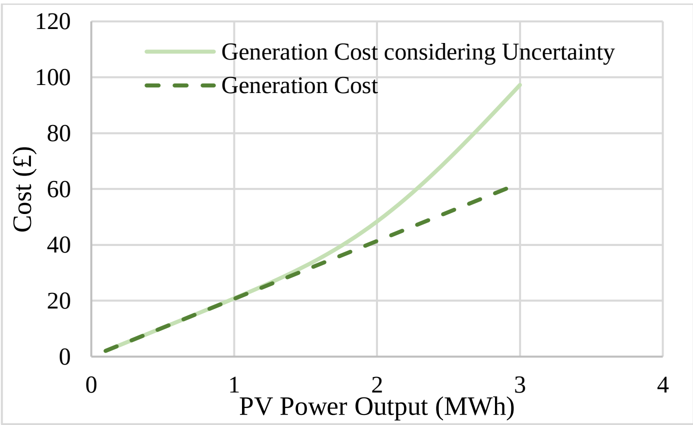

Fig. 8. Generation cost for the PV power station

TABLE III

OPTIMAL POWER OUTPUT FOR SUPPLIERS

Supplier		Thermal	Wind	PV	Storage
*P** (MWh)	Without uncertainty cost	14.5	18.2	3	3.2
	With uncertainty cost	17	11.7	1.6	5.8

Table III compares the four suppliers' optimal generation strategies with and without the consideration of renewable uncertainty cost. It is clear that, with such uncertainty cost, the optimal generation for the wind plant reduces by about a third, from 18.2MWh to 11.7MWh. Compared to wind power, the power output for PV falls from 3MWh to 1.6MWh, nearly being cut in a half. In this case, the PV output C~(2, 0.5) has more fluctuations than the wind power N~(15,2), thus highly affected by the charge of uncertainty cost, in accordance with the results. By considering the uncertainty cost, renewable suppliers would reduce their planned supply to mitigate the risk of suffering an expensive shortage penalty. On the other hand, the thermal power station and controllable storage supplier are more competitive so that their optimal generation strategies have an increase of 2.6MWh and 2.7MWh.

5.3 Optimal Strategies with EV Aggregator Included

In this case, an EV aggregator is also introduced into the local energy market trading. It is assumed that EV aggregator operates 100 EVs under 95% charging efficiency and the capacity is 50kwh for each EV. The charging price and battery degradation cost for each car are set at £16/MWh and £0.1. In practice, it is possible that not all EVs signed up for trading can be immediately available to charge as scheduled the aggregator, and thus the PDF of the number of available EVs is assumed as the normalized distribution (50,30). Based on the proposed OGP algorithm, the optimal strategies of these five suppliers in the an-hour period are derived at the NE point. With this strategy, each supplier could maximize its revenue. Their revenues and optimal strategies are demonstrated in Fig.9.

$$\{P_t^*, P_w^*, P_{PV}^*, P_s^*, P_{EV}^*\} = \{14, 12.5, 1.6, 2.7, 3.2\}\text{MWh}$$

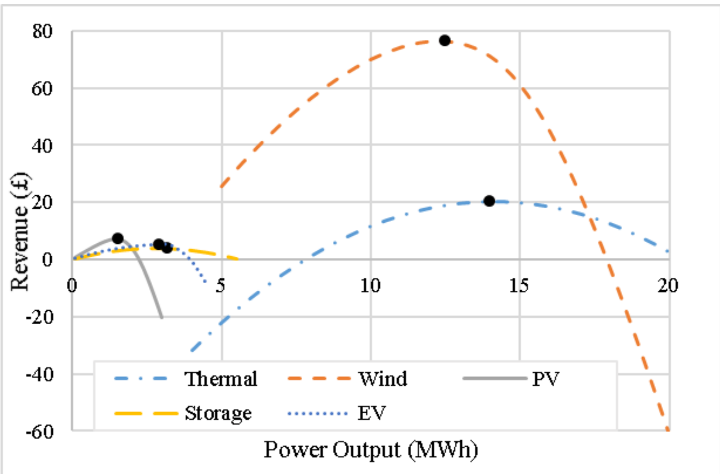

Fig. 9 Revenue for five suppliers when the other four suppliers all follow the optimal generation

Their maximum revenue points are marked in black points. For the thermal power station, initially, the profit is too low to cover its generation cost, leading to a loss before 8MWh output. Then, it achieves profits with the growing output and realizes the maximum revenue of £20 at 14MW, matching the NE point. For wind, PV, and EV aggregators, their revenue curves show a similar convex trend. In growing production, their revenues gradually increase to the maximum points, respectively £76.4 at 12.5MWh, £7 at 1.6MWh, and £5.1 at 3.2MWh, and then fall to the deficits which are caused by expensive uncertainty costs. During high production periods, more power required means more potential risk. It clearly shows that for these three suppliers, the uncertainty costs are unaffordable when the outputs are over 18MWh, 2.2MWh, and 4MWh separately. For storage, it arrives at the maximum profitable point of £3.9 with an output of 2.7MWh. All suppliers realize their maximum revenues at the NE point, which proves the effectiveness of the proposed approach.

By summarising the above case studies, this proposed method has the following benefits to key stakeholders:

- For energy customers, the proposed model efficiently guides suppliers to play in the market, and thus customers can buy cheaper energy locally compared to that from the main energy market. In this way, customers can also avoid paying high network costs as their use of the main grid is reduced. They can also have a more secure supply once there outages in the main grid by procuring energy locally.
- For society, the proposed local energy trading for suppliers in the local energy market can promote the local generation and supplier business in energy transactions, which will help local businesses. In addition, the flourishing local energy market not only enhances supply security and releases the constraints on the main grid, but also provides local suppliers and customers more options to maximize their profits.
- For the environment, the proposed method can promote local balancing, so that renewable energy can be consumed locally as much as possible. This will help reduce emissions, as normally renewable energy has to be curtailed due to its intermittency or constrained network.

Due to transportation constraints, it is possible that not all EVs signed up for trading can be immediately available to charge as scheduled the aggregator. The energy shortage can cause trading risk for the aggregator who has to purchase that from the ancillary market to ensure the transaction, which is modelled as the uncertainty cost for the aggregator. It is thus assumed that the number of available EVs for discharging is stochastic in the trading. During high load periods, more energy is required and thus the higher risk of shortage is induced, which could lead to the expensive uncertainty costs for EV aggregators. During low load periods, the risks tend to be low due to both low energy prices and total trading amount. The impact of uncertainty is also studied in the case study. EV aggregator's revenue curve shows a convex trend. With growing demand, the revenue gradually increases to the maximum of £5.1 at 3.2MWh, and then falls to the deficits in high demand periods caused by the expensive uncertainty costs.

6. CONCLUSIONS

This paper proposes a Cournot oligopoly model in the local energy market to formulate the competition among non-cooperative suppliers with consideration of renewable uncertainty. Different from most current research without differentiating the features of multiple suppliers (traditional, renewable, storage, etc.), this paper designs individual business models that effectively derive the optimal generation strategy to benefit suppliers in local energy trading. Through extensive demonstration, the following key observations are obtained:

- Four typical electricity providers – thermal power station, wind power generation, PV power generation, and electricity storage are analysed respectively. In addition, EV aggregators are considered as well. Their individual business models are formulated in mathematical models created in mathematics to formulate their trading behaviours and the competitive strategies are concluded for guidance.

- This paper studies renewables uncertainty and models it in energy trading, which promotes DERs in the local market. The uncertainties are formulated in production and the concept of uncertainty cost is presented to weigh their generation capacity and provide security for energy trading. In addition, it is shown in the case study that, with this charge applied, these renewables generations would prefer conservative strategies, which properly reduce their production to guarantee an adequate supply. Meanwhile, it improves the stability of the local energy market and enhances the security of the power system.
- In the supplier competition model, the price elasticity of demand is applied for price constraint to deal with the complex interactions between demand, supply, and price signals, which reflects the rigid demand and elastic demand in daily electricity consumption. This point is combined in Cournot Oligopoly to make the model more practical.
- The OGP Algorithm is to find the NE point that effectively derives the equilibrium solutions. The participated suppliers could be benefited by using these optimal generation strategies to gain the maximum profits in local energy trading. With this work facilitated in the local energy market, it enhances the local energy trading and promotes the application of DER to relieve the heavy burden of the conventional power system.

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